

# The Effect of the Field Excited by Beam on the $\Delta t$ Measurement

LU-231

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Jan. 1994

## 1 The Electric Field Excited by Beam

The electric field excited by beam with synchrotron  $\beta$  in a resonant tank is given by<sup>[1]</sup>:

$$E_{b,s} \doteq -Z_{eff} I_o F_1 K_1 \exp j(\omega t - \varphi_b) \{1 - \exp[-(t - t_o)/\tau]\} \cos(k_1 z) \quad (1)$$

where  $F_1=1$ , in our case typically  $Z_{eff} \doteq 40 \text{ M}\Omega/\text{m}$ ,  $K_1 = \frac{Q_{\text{loaded}}}{Q_{\text{unloaded}}} \doteq 1/2$ ,  $\tau \doteq 15 \mu\text{s}$ , thus for beam current of  $I_o=30 \text{ mA}$ , pulse duration  $t - t_o = 30 \mu\text{s}$ ,  $E_{b,s} \doteq 0.52 \text{ MV/m}$ . This is in good agreement with the measured result.

When we use the program of beam phase scan or  $\Delta t$  measurement, all the theory assume that the beam is drifting in a field-free space in tank-N and tank-N+1, as shown in Fig.1, when tank-N is turn-off. However an electric field may be excited in tank-N by the beam. When the beam is turn-off in tank-N, the beam is not synchrotron with the geometric  $\beta$ , since it is not accelerated, thus the excited field will be much smaller when the injecting  $\beta$  is far from the geometric  $\beta$ . Assuming that the injecting  $\beta$  is constant in tank-N, the order of phase difference  $\Delta\varphi_d$  relative to the injection phase at the output of the tank-N is estimated as follows:

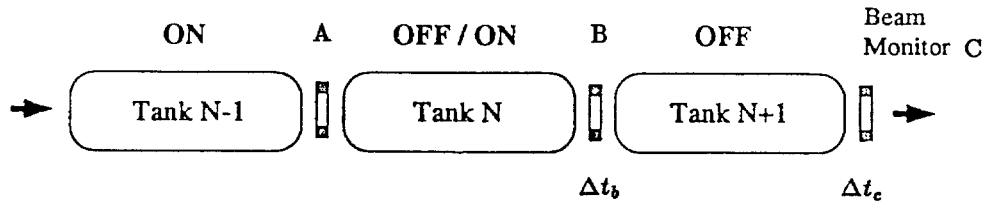


Figure 1: The scheme of Phase-scan or  $\Delta t$  measurement

Tank	$\beta$	Drift length (m)	Drift time (ns)	$\Delta\varphi_d(^{\circ})$
11	0.4569042	6.57279	47.98	764
12	0.5093977	7.2414838	47.417	601
13	0.5553564	7.8269944	47.01	483
14	0.5956159	8.3400689	46.706	394.8
15	0.6309325	8.7911907	46.476	328.3
16	0.6620435	9.1885148	46.294	275.5
17	0.6894767	9.5399603	46.1525	234.4

It is difficult to estimate the excited field theoretically. Physically, it may be expected that the greater the  $\Delta\varphi_d$ , the lower the excited field; or the higher the beam energy, the closer to the geometric  $\beta$  the injecting  $\beta$  is and the higher the excited field. It is estimated by measurement that the excited field in tank-16 is  $\sim 0.0125E_o$ , when the tank-16 is turn-off, where  $E_o$  is the normal operational field amplitude. In this case, the resulted measurement error  $\delta\varphi_{ab}$  in  $\Delta\varphi_{ab}$  (using the detector after tank-16) and  $\delta\varphi_{ac}$  in  $\Delta\varphi_{ac}$  (using the detector after tank-17) are shown in following table, when the tank-16 is off, assuming the field in tank-17 is zero.

$\varphi_{\text{input}}$ ( $^{\circ}$ ) in 805 MHz	$\delta\varphi_{ab}$ ( $^{\circ}$ ) in 201 MHz	$\delta\varphi_{ac}$	$\beta_{\text{output}}$	$\delta W$ (MeV)
-1.08000E+03	0.122081E+00	0.111290E-01	0.662028E+00	-0.0236
-9.60000E+02	-.307497E-01	-.280769E+00	0.662002E+00	-0.0620
-.840000E+02	-.184121E+00	-.561378E+00	0.661979E+00	-0.0959
-.720000E+02	-.331252E+00	-.818692E+00	0.661959E+00	-0.1254
-.600000E+02	-.465682E+00	-.104182E+01	0.661943E+00	-0.1490
-.480000E+02	-.581558E+00	-.122139E+01	0.661931E+00	-0.1667
-.360000E+02	-.673889E+00	-.134989E+01	0.661924E+00	-0.177
-.240000E+02	-.738744E+00	-.142195E+01	0.661923E+00	-0.1785
-.120000E+02	-.773415E+00	-.143452E+01	0.661927E+00	-0.1727
0.000000E+00	-.776503E+00	-.138702E+01	0.661936E+00	-0.1594
0.120000E+02	-.747969E+00	-.128134E+01	0.661951E+00	-0.1372
0.240000E+02	-.689114E+00	-.112181E+01	0.661969E+00	-0.1107
0.360000E+02	-.602513E+00	-.915062E+00	0.661991E+00	-0.0782
0.480000E+02	-.491908E+00	-.669794E+00	0.662015E+00	-0.0428
0.600000E+02	-.362046E+00	-.396433E+00	0.662042E+00	-0.0030
0.720000E+02	-.218487E+00	-.106735E+00	0.662068E+00	0.0354
0.840000E+02	-.673843E-01	0.186698E+00	0.662094E+00	0.0738
0.960000E+02	0.847683E-01	0.470960E+00	0.662119E+00	0.1107

In conclusion, if the measured result is correct, the measurement error in  $t_{\text{off}}$  for tank-16 is  $\delta\varphi_{ab} \sim 2\text{--}3^{\circ}$  when the detector-16 is used;  $\delta\varphi_{ac} \sim 4\text{--}5.6^{\circ}$  when the detector-17 is used. The change in beam energy is  $\sim 0.17$  MeV. That means this effect is still not negligible for an accurate measurement and a high energy accelerator.

## 2 The Effect on the $\Delta t$ Measurement

The measured quantity in  $\Delta t$  measurement is:

$$\Delta t_{ab} = t_{ab_{off}} - t_{ab_{on}} \quad (2)$$

$$\Delta t_{ac} = t_{ac_{off}} - t_{ac_{on}} \quad (3)$$

If the field compensation in tank-16 is complete when tank-N is turn-on, the measurement error in  $\Delta t_{ab}$  is just the error in  $t_{ab_{off}}$ . So there is an error of  $\delta\varphi_{ab} \sim 2-3^\circ$  in the abscissa of the  $\Delta t$  surface. For the detector-17, except the error in  $t_{ac_{off}}$  there is an error in  $t_{ac_{on}}$  which is resulted from the beam exciting field in tank-17. Assuming the beam exciting field in tank-17 is also about  $-0.0125E_o$ , when tank-16 is on, the error in  $t_{ac_{on}}$  is about  $2.4^\circ$ . When the phase in tank-16 changes, the output  $\beta$  changes. This will result in different error, but this influence is negligible ( $\sim 0.1^\circ$  for  $\varphi_s \pm 10^\circ$ ). That is said there is also an error of  $\delta\varphi_{ac} \sim 2-3^\circ$  in the ordinate of the  $\Delta t$  surface. Both error can be treated as a constant around the synchrotron phase.

In  $\Delta t$  measurement<sup>[2]</sup>, it is determined by measurement :

$$\Delta\varphi_{ab} = k_1\varphi + b_1 \quad (4)$$

$$\Delta\varphi_{ac} = k_2\varphi + b_2 \quad (5)$$

A constant error will result in an error in the constant  $b_1$  and  $b_2$ . The operational field amplitude is determined by the slope of  $k_2/k_1$ . Thus there is little influence by this effect. The synchrotron phase is determined by:

$$\varphi_s = -\frac{b_1 a_{11} + b_2 a_{12}}{k_1 a_{11} + k_2 a_{12}} \quad (6)$$

or

$$\varphi_s = -\frac{b_2 a_{11} - b_1 a_{12}}{k_2 a_{11} - k_1 a_{12}} \quad (7)$$

where  $a_{ij}$  is the following matrix coefficient:

$$\begin{pmatrix} \Delta\varphi_A \\ \Delta W_A \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \Delta\varphi_{ab} \\ \Delta\varphi_{ac} \end{pmatrix} \quad (8)$$

Thus the resulted error in  $\varphi_s$  is about:

$$\delta\varphi_s \doteq -\frac{\delta\varphi_{ab}a_{11} + \delta\varphi_{ac}a_{12}}{k_1 a_{11} + k_2 a_{12}} \quad (9)$$

or

$$\delta\varphi_s \doteq -\frac{\delta\varphi_{ac}a_{11} - \delta\varphi_{ab}a_{12}}{k_2 a_{11} - k_1 a_{12}} \quad (10)$$

The input energy is determined by:

$$\Delta W_A = \frac{a_{22}}{k_1}(b_2 k_1 - b_1 k_2) \quad (11)$$

Thus the error in input energy is about:

$$\delta W_A \doteq \frac{a_{22}}{k_1}(\delta\varphi_{ac}k_1 - \delta\varphi_{ab}k_2) \quad (12)$$

### 3 The Effect on the Phase-scan Measurement

Only one detector is used in Phase-scan measurement. When the detector-16 is used, the error in the ordinate of the phase-scan curve is just the  $\delta\varphi_{ab}$  when the tank-16 is off. That is said the total curve is moved up by a constant quantity of  $\delta\varphi_{ab} \approx 2\text{—}3^\circ$ . When the detector-17 is used, the error in the ordinate of the phase-scan curve is:

$$\delta\varphi = \delta\varphi_{ac_{off}} - \delta\varphi_{bc_{on}} \quad (13)$$

where  $\delta\varphi_{ac_{off}}$  is the drift time error in tank-16 and tank-17 when the tank-16 is off,  $\delta\varphi_{bc_{on}}$  is the drift time error in tank-17 when the tank-16 is on. As known,  $\delta\varphi_{ac_{off}} \approx 4\text{—}5.6^\circ$ , which is a constant. Since the input energy in tank-17 is different much when the tank-16 is turn-on at different phase. The higher the input energy, the larger the  $\delta\varphi_{bc_{on}}$ , and the smaller the  $\delta\varphi$ . That is said in the region around the synchrotron phase the error is smaller, about  $2\text{—}3^\circ$ . In the region around low energy the error is greater, about  $3\text{—}4.6^\circ$ .

In conclusion, the effect of the field excited by beam on the  $\Delta t$  measurement is still required to be studied for high energy accelerator.

#### REFERENCES

- [1] P.Lapostolle, "Linear Accelerator", (1970).
- [2] P.N.Ostroumov, "USING DELTA-T IN FERMILAB LINAC", Oct.12,1993.